CS 163 Discrete Math **Fall 2023**
http://neilklingensmith.com/teaching/loyola/cs163/

Practice Midterm

Date: October 16, 2023

Name:

1. (24 points) Convert the follwoing numbers in decimal representation to binary and hexadecimal:

Decimal Representation	Binary Representation	Hexadecimal Representation
29	1 1101	0x1D
127	111 1111	0x7F
84	101 0100	0x54
32	10 0000	0x20

2. (20 points) Compute the follwoing modulus (*Hint: first convert to binary.*)
   (a) (5 points) 29 MOD 8

   Solution: $29 = 1\ 1101_2$
   $1\ 1101_2 \text{ MOD } 8 = 101_2 = 5_{10}$

   (b) (5 points) 127 MOD 16

   Solution: $127 = 111\ 1111_2$
   $111\ 1111_2 \text{ MOD } 16 = 1111_2 = 15_{10}$

   (c) (5 points) 84 MOD 4

   Solution: $84 = 0101\ 0100_2$
   $0101\ 0100_2 \text{ MOD } 4 = 00_2 = 0_{10}$

   (d) (5 points) 32 MOD 16

   Solution: $32 = 10\ 0000_2$
   $10\ 0000_2 \text{ MOD } 16 = 0_2 = 0_{10}$

3. (50 points) **Arithmetic on an 16-bit processor.** We have a really $#!tty 16-bit processor that only has an adder and a bit shifter. It has no ability to perform multiplication or division. We need to compute $(5011_{10} - 2899_{10})/4$ using only addition and bit shifts.

   (a) (15 points) First calculate the 2's complement representation of -2899. In the box below, write out the binary representation of $+2899$, then take its two's complement. Also convert the binary to hex in the boxes at right.

	Binary	Hex
$+2899$		0x
1's(2899)		0x
2's(2899)		0x

   (b) (15 points) Now add the two's complement of 2899 to 5011. The result should be the same as 5011-2899.

	Binary	Hex
2's(2899)		0x
5011_{10}		0x
$2's(2899) + 5011_{10}$		0x

   (c) (10 points) Now divide the result of the addition from part 3(b) by 4 using a bit shift.

	Binary	Hex
$(2's(2899) + 5011_{10})/4$		0x

   (d) (10 points) Convert the result from part 3(c) to **decimal**.

   Solution: ????

4. (20 points) Suppose we have some binary number X that consists of three bits: $X_2X_1X_0$. Write a logic function that is true under the following conditions:

   (a) (5 points) X contains only one 1.

   Solution: $(X_2 \,\&\, \overline{X_1} \,\&\, \overline{X_0}) \mid (\overline{X_2} \,\&\, X_1 \,\&\, \overline{X_0}) \mid (\overline{X_2} \,\&\, \overline{X_1} \,\&\, X_0)$
   A slightly simpler version:
   $\overline{X_2} \,\&(X_1 \oplus X_0) \mid (X_2 \,\&\, \overline{X_1} \,\&\, \overline{X_0})$

   (b) (5 points) X contains an even number of 1s.

   Solution: $\overline{X_2 \oplus X_1 \oplus X_0}$

   (c) (5 points) X when interpreted as an unsigned binary number is less than 3.

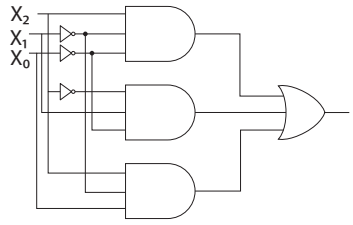
   Solution: $\overline{X_2} \,\&\, \overline{(X_1 \,\&\, X_0)}$

   (d) (5 points) X when interpreted as a signed (2's complement) number is less than -1.

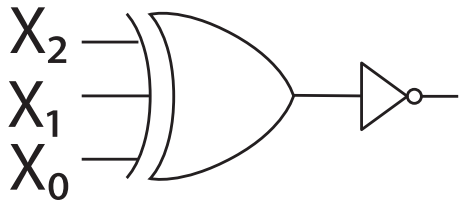
   Solution: $X_2 \,\&\, \overline{(X_1 \,\&\, X_0)}$

5. (20 points) Implement the four logic functions from Question 4 using gates.

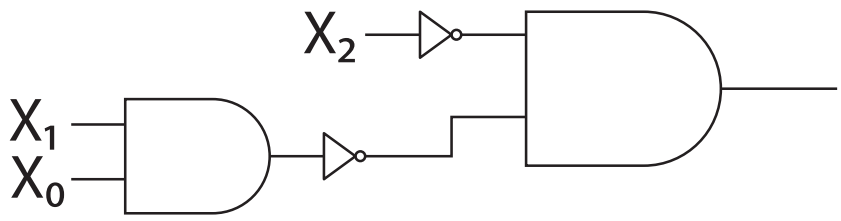
   (a) (5 points) **Solution:**



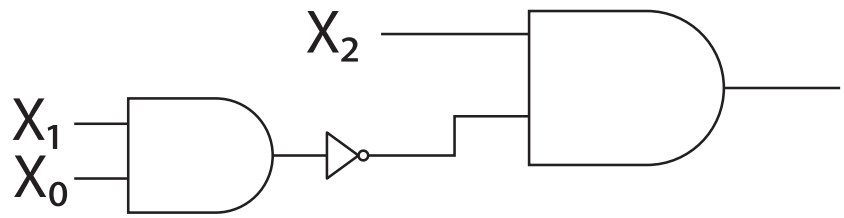
   (b) (5 points) **Solution:**



   (c) (5 points) **Solution:**



(d) (5 points) **Solution:**



6. (40 points) Implement a switching network that has two data inputs (A and B), two data outputs (C and D), and a control input S. When S = 1, the network is in passthrough mode, and C should equal A, and D should equal B. If S equals 0, the network is in crossing mode, and C should equal B and D should equal A.