**CS 163 Discrete Math** **Fall 2019**
http://neilklingensmith.com/teaching/loyola/cs163/

# Quiz 4 Study Guide

Date: October 3, 2019

**Name:**

1. (40 points) Dungeons and Dragons: When you roll a (fair) 10-sided die, there are ten possible outcomes.
   (a) (10 points) If the box below represents the sample space $\Omega$, draw and label all the possible outcomes.

$\Omega$

| | |
|---|---|
| 1 | 2 |
| 3 | 4 |
| 5 | 6 |
| 7 | 8 |
| 9 | 10 |

   (b) (10 points) In the sample space above, highlight the event that the roll is less than seven.
   (c) (10 points) What is the probability of rolling less than 7? Show your work, don't just write down a number.

**Solution:**

$$P(\text{Less than 7}) = P(1) + P(2) + P(3) + P(4) + P(5) + P(6) = \frac{6}{10}$$

   (d) (10 points) In the dice rolling example, let $E_1$ be the event that you roll a 1, 2 or a 3 and let $E_2$ be the event that your roll a 1 or a 4. Calculate $P(E_1 \cup E_2)$.

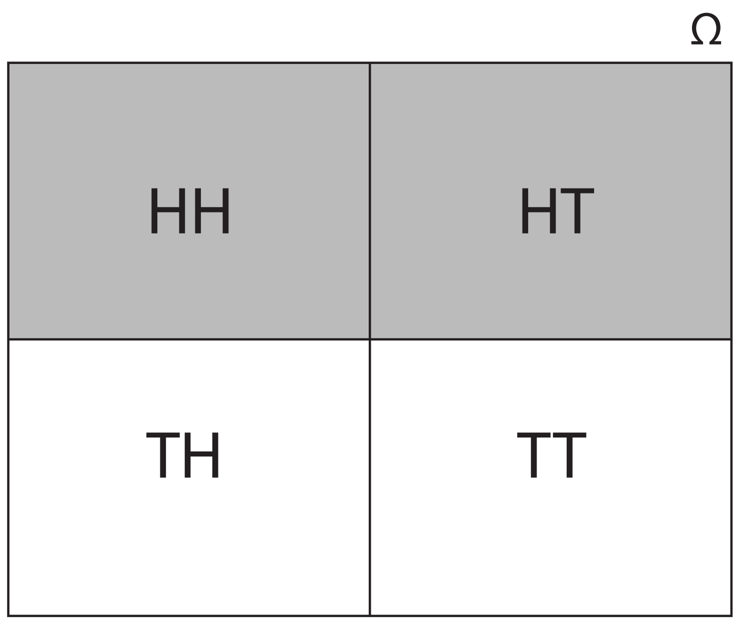
**Solution:**

$$E_1 = \{1, 2, 3\} E_2 = \{1, 4\}$$

Since there is overlap between $E_1$ and $E_2$, we cannot just add their individual probabilities together because if we did, we'd be double-counting the overlap. The overlap is $E_1 \cap E_2 = \{1\}$. We have to subtract the probability of the overlap from the sum. $P(E_1 \cap E_2) = P(\{1\}) = \frac{1}{10}$.

$$P(E_1 \cup E_2) = P(E_1) + P(E_2) - P(E_1 \cap E_2) = \frac{3}{10} + \frac{2}{10} - \frac{1}{10} = \frac{4}{10}$$

2. (20 points) Conditional Probability




(a) (5 points) If you flip a fair coin twice, draw all the possible outcomes in the sample space above.

(b) (5 points) What is the probability of getting two heads? We will call this event $E_1 = \{HH\}$.

**Solution:**
$$P(\{HH\}) = \frac{1}{4}$$

(c) (5 points) Now we're going to run the coin flipping experiment again, but this time, suppose an oracle tells us that the first flip will be heads. Highlight the region in the sample space for which the first flip is heads. We will call this event $E_2$, the event that the first flip is heads.

**Solution:** Formally, $E_2$ can be written as a set of outcomes:
$$E_2 = \{HH, HT\}$$

(d) (5 points) What is the conditional probability $P(E_1|E_2)$? This is the probability that the sequence of flips will be $\{HH\}$ given that the first flip was heads.

**Solution:** We will use the formula for conditional probability: $P(E_1|E_2) = \frac{P(E_1 \cap E_2)}{P(E_2)}$.
$$E_1 = \{HH\}$$
$$E_2 = \{HH, HT\}$$

$$E_1 \cap E_2 = \{HH\} \cap \{HH, HT\} = \{HH\}$$

$$P(E_1|E_2) = \frac{P(E_1 \cap E_2)}{P(E_2)} = \frac{1/4}{1/2} = \frac{1}{2}$$