79	162	Discrete	Math
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Fall 2019

http://neilklingensmith.com/teaching/loyola/cs163/

Midterm A

Date: October 17, 2019

Name:

1. (16 points) In the following list of functions, circle the properties that apply to each.

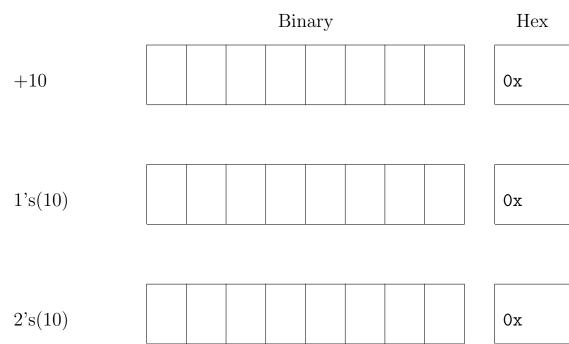
f(x) = -x	linear	onto	one-to-one	bijective
$f(x) = x^3$	linear	onto	one-to-one	bijective
$f(x) = \sqrt{x}$	linear	onto	one-to-one	bijective
$f(x) = \sum_{i=1}^{N} a_i \times x$	linear	onto	one-to-one	bijective

2. (10 points) How many possible sequences of Heads and Tails are there in five coin flips?

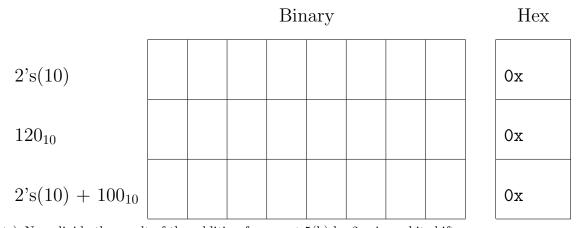
3. (9 points) Consider the set $A = \{1, 3, 5, 7, 9\}$ with card(A) = k. What is $card(\mathcal{P}(A))$?

4. (10 points) How many ways are there to choose 3 balls out of a set of 5?

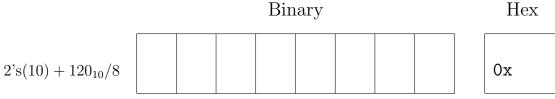
- 5. (50 points) Arithmetic on an 8-bit processor. We have a really #!tty 8-bit processor that only has an adder and a bit shifter. It has no ability to perform multiplication or division. We need to compute $(120_{10} 10_{10})/8$ using only addition and bit shifts.
 - (a) (15 points) First we're going to calculate the 2's complement representation of -10. In the box below, write out the binary representation of +10, then take its two's complement. Also convert the binary to hex in the boxes at right.



(b) (15 points) Now add the two's complement of 10 to 120. The result should be the same as 120-10.



(c) (10 points) Now divide the result of the addition from part 5(b) by 2 using a bit shift.



(d) (10 points) Convert the result from part 5(c) to **decimal**.

6. (10 points) **How to cheat on Draft Kings.** Below is a table of stats for Colin Kaepernick (49ers QB) for the 2012 season.

Week	Game Date	Opponent	Result	Num Sacks	Fumbles			
4	9/5	Jets	Win, 34-0	0	0			
5	9/15	Bills	Win, $45-3$	0	1			
6	9/22	Giants	Loss $3-26$	2	0			
10	9/26	Rams	Tie, 24-24	3	2			
11	10/6	Bears	Win, 32-7	1	0			
12	10/6	Saints	Win, 31-21	0	0			
13	10/6	Rams	Loss, 13-16	3	1			
14	10/6	Dolphins	Win, 27-13	4	1			
15	10/6	Patriots	Win, 41-34	1	4			
16	10/6	Seahawks	Loss, 13-42	1	0			
17	10/6	Cardinals	Win, 27-13	1	0			
Note: there's no week 18. Break btw regular season and postseason.								
19	10/6	Packers	Win, 45-31	1	1			
20	10/6	Falcons	Win, 28-24	1	0			
22	10/6	Ravens	Loss, $31-34$	3	0			

(a) (5 points) Based on this data, what is the overall probability that the 49ers will win a game this season?

(b) (15 points) What is the conditional probability that the 49ers will win the next game given that Kaepernick is sacked in the current game? *Hint: there are only four pairs of sequential games in this data.*

(c) (15 points) What is the conditional probability that the 49ers will win the next game given that Kaepernick is sacked **and** he fumbles in the current game? *Hint: there are only four pairs of sequential games in this data.*

(d) (20 points) Based on the info in this table, compute the probability that the 49ers will win the next game. Hint: your calculation should be something like $Pr(win|sack \ and \ no \ fumble)$

- (e) (10 points) Consider the following events:
 - E_1 The event that the 49ers win the next game
 - E_2 The event that Kaepernick fumbles in the current game
 - Is E_1 independent of E_2 ? Explain your reasoning with some math or a formula. Hint: what is the definition of statistical independence?