

Midterm A

Date: October 17, 2019

Name:

1. (16 points) In the following list of functions, circle the properties that apply to each.

$f(x) = -x$ linear onto one-to-one bijective

$f(x) = x^3$ linear onto one-to-one bijective

$f(x) = \sqrt{x}$ linear onto one-to-one bijective

$f(x) = \sum_{i=1}^N a_i \times x$ linear onto one-to-one bijective

2. (10 points) How many possible sequences of Heads and Tails are there in five coin flips?

3. (9 points) Consider the set $A = \{1, 3, 5, 7, 9\}$ with $\text{card}(A) = k$. What is $\text{card}(\mathcal{P}(A))$?

4. (10 points) How many ways are there to choose 3 balls out of a set of 5?

5. (50 points) **Arithmetic on an 8-bit processor.** We have a really \$#!tty 8-bit processor that only has an adder and a bit shifter. It has no ability to perform multiplication or division. We need to compute $(120_{10} - 10_{10})/8$ using only addition and bit shifts.

(a) (15 points) First we're going to calculate the 2's complement representation of -10 . In the box below, write out the binary representation of $+10$, then take its two's complement. Also convert the binary to hex in the boxes at right.

	Binary	Hex									
+10	<table border="1" style="width: 100%; height: 25px; border-collapse: collapse;"> <tr> <td style="width: 12.5%;"></td> <td style="width: 12.5%;"></td> <td style="width: 12.5%;"></td> <td style="width: 12.5%;"></td> <td style="width: 12.5%;"></td> <td style="width: 12.5%;"></td> <td style="width: 12.5%;"></td> <td style="width: 12.5%;"></td> </tr> </table>									<table border="1" style="width: 100%; height: 40px; border-collapse: collapse;"> <tr> <td style="text-align: center; vertical-align: middle;">0x</td> </tr> </table>	0x
0x											
1's(10)	<table border="1" style="width: 100%; height: 25px; border-collapse: collapse;"> <tr> <td style="width: 12.5%;"></td> <td style="width: 12.5%;"></td> <td style="width: 12.5%;"></td> <td style="width: 12.5%;"></td> <td style="width: 12.5%;"></td> <td style="width: 12.5%;"></td> <td style="width: 12.5%;"></td> <td style="width: 12.5%;"></td> </tr> </table>									<table border="1" style="width: 100%; height: 40px; border-collapse: collapse;"> <tr> <td style="text-align: center; vertical-align: middle;">0x</td> </tr> </table>	0x
0x											
2's(10)	<table border="1" style="width: 100%; height: 25px; border-collapse: collapse;"> <tr> <td style="width: 12.5%;"></td> <td style="width: 12.5%;"></td> <td style="width: 12.5%;"></td> <td style="width: 12.5%;"></td> <td style="width: 12.5%;"></td> <td style="width: 12.5%;"></td> <td style="width: 12.5%;"></td> <td style="width: 12.5%;"></td> </tr> </table>									<table border="1" style="width: 100%; height: 40px; border-collapse: collapse;"> <tr> <td style="text-align: center; vertical-align: middle;">0x</td> </tr> </table>	0x
0x											

(b) (15 points) Now add the two's complement of 10 to 120. The result should be the same as $120 - 10$.

	Binary	Hex									
2's(10)	<table border="1" style="width: 100%; height: 25px; border-collapse: collapse;"> <tr> <td style="width: 12.5%;"></td> <td style="width: 12.5%;"></td> <td style="width: 12.5%;"></td> <td style="width: 12.5%;"></td> <td style="width: 12.5%;"></td> <td style="width: 12.5%;"></td> <td style="width: 12.5%;"></td> <td style="width: 12.5%;"></td> </tr> </table>									<table border="1" style="width: 100%; height: 40px; border-collapse: collapse;"> <tr> <td style="text-align: center; vertical-align: middle;">0x</td> </tr> </table>	0x
0x											
120 ₁₀	<table border="1" style="width: 100%; height: 25px; border-collapse: collapse;"> <tr> <td style="width: 12.5%;"></td> <td style="width: 12.5%;"></td> <td style="width: 12.5%;"></td> <td style="width: 12.5%;"></td> <td style="width: 12.5%;"></td> <td style="width: 12.5%;"></td> <td style="width: 12.5%;"></td> <td style="width: 12.5%;"></td> </tr> </table>									<table border="1" style="width: 100%; height: 40px; border-collapse: collapse;"> <tr> <td style="text-align: center; vertical-align: middle;">0x</td> </tr> </table>	0x
0x											
2's(10) + 100 ₁₀	<table border="1" style="width: 100%; height: 25px; border-collapse: collapse;"> <tr> <td style="width: 12.5%;"></td> <td style="width: 12.5%;"></td> <td style="width: 12.5%;"></td> <td style="width: 12.5%;"></td> <td style="width: 12.5%;"></td> <td style="width: 12.5%;"></td> <td style="width: 12.5%;"></td> <td style="width: 12.5%;"></td> </tr> </table>									<table border="1" style="width: 100%; height: 40px; border-collapse: collapse;"> <tr> <td style="text-align: center; vertical-align: middle;">0x</td> </tr> </table>	0x
0x											

(c) (10 points) Now divide the result of the addition from part 5(b) by 2 using a bit shift.

	Binary	Hex									
2's(10) + 120 ₁₀ /8	<table border="1" style="width: 100%; height: 25px; border-collapse: collapse;"> <tr> <td style="width: 12.5%;"></td> <td style="width: 12.5%;"></td> <td style="width: 12.5%;"></td> <td style="width: 12.5%;"></td> <td style="width: 12.5%;"></td> <td style="width: 12.5%;"></td> <td style="width: 12.5%;"></td> <td style="width: 12.5%;"></td> </tr> </table>									<table border="1" style="width: 100%; height: 40px; border-collapse: collapse;"> <tr> <td style="text-align: center; vertical-align: middle;">0x</td> </tr> </table>	0x
0x											

(d) (10 points) Convert the result from part 5(c) to **decimal**.

6. (10 points) **How to cheat on Draft Kings.** Below is a table of stats for Colin Kaepernick (49ers QB) for the 2012 season.

Week	Game Date	Opponent	Result	Num Sacks	Fumbles
4	9/5	Jets	Win, 34-0	0	0
5	9/15	Bills	Win, 45-3	0	1
6	9/22	Giants	Loss 3-26	2	0
10	9/26	Rams	Tie, 24-24	3	2
11	10/6	Bears	Win, 32-7	1	0
12	10/6	Saints	Win, 31-21	0	0
13	10/6	Rams	Loss, 13-16	3	1
14	10/6	Dolphins	Win, 27-13	4	1
15	10/6	Patriots	Win, 41-34	1	4
16	10/6	Seahawks	Loss, 13-42	1	0
17	10/6	Cardinals	Win, 27-13	1	0
<i>Note: there's no week 18. Break btw regular season and postseason.</i>					
19	10/6	Packers	Win, 45-31	1	1
20	10/6	Falcons	Win, 28-24	1	0
22	10/6	Ravens	Loss, 31-34	3	0

- (a) (5 points) Based on this data, what is the overall probability that the 49ers will win a game this season?
- (b) (15 points) What is the conditional probability that the 49ers will win the next game given that Kaepernick is sacked in the current game? *Hint: there are only four pairs of sequential games in this data.*
- (c) (15 points) What is the conditional probability that the 49ers will win the next game given that Kaepernick is sacked **and** he fumbles in the current game? *Hint: there are only four pairs of sequential games in this data.*
- (d) (20 points) Based on the info in this table, compute the probability that the 49ers will win the next game. *Hint: your calculation should be something like $Pr(\text{win}|\text{sack and no fumble})$*

(e) (10 points) Consider the following events:

E_1 The event that the 49ers win the next game

E_2 The event that Kaepernick fumbles in the current game

Is E_1 independent of E_2 ? Explain your reasoning with some math or a formula. *Hint: what is the definition of statistical independence?*