CS 163 Discrete Math **Fall 2019**
http://neilklingensmith.com/teaching/loyola/cs163/

Practice Midterm

Date: October 17, 2019

Name:

1. (16 points) In the following list of functions, circle the properties that apply to each. For all functions, assume that the domain and the co-domain are $\mathbb{R}$, the set of reals (positive and negative).

$f(x) = -x$	linear	onto	one-to-one	bijective
$f(x) = x^3$	linear	onto	one-to-one	bijective
$f(x) = \sqrt{x}$	linear	onto	one-to-one	bijective
$f(x) = \sum_{i=1}^{N} a_i \times x$	linear	onto	one-to-one	bijective

2. (10 points) How many possible sequences of Heads and Tails are there in five coin flips?

Solution: $2^5 = 32$

3. (9 points) Consider the set $A = \{1, 3, 5, 7, 9\}$. What is $card(\mathcal{P}(A))$?

Solution: $2^5 = 32$

4. (10 points) How many ways are there to choose 3 balls out of a set of 5?

Solution:

$$\binom{5}{3} = \frac{5!}{3!(5-3)!} = 10$$

5. (50 points) **Arithmetic on an 8-bit processor.** We have a really $#!tty 8-bit processor that only has an adder and a bit shifter. It has no ability to perform multiplication or division. We need to compute $(120_{10} - 10_{10})/8$ using only addition and bit shifts.

   (a) (15 points) First we're going to calculate the 2's complement representation of -10. In the box below, write out the binary representation of $+10$, then take its two's complement. Also convert the binary to hex in the boxes at right.

	Binary								Hex
+10									0x
1's(10)									0x
2's(10)									0x

(b) (15 points) Now add the two's complement of 10 to 120. The result should be the same as 120-10.

	Binary								Hex
2's(10)									0x
120_{10}									0x
2's(10) + 100_{10}									0x

(c) (10 points) Now divide the result of the addition from part 5(b) by 2 using a bit shift.

	Binary								Hex
2's(10) + $120_{10}/8$									0x

(d) (10 points) Convert the result from part 5(c) to **decimal**.

Solution: ????

6. (10 points) **How to cheat on Draft Kings.** Below is a table of stats for Colin Kaepernick (49ers QB) for the 2012 season.

Week	Game Date	Opponent	Result	Num Sacks	Fumbles
4	9/5	Jets	Win, 34-0	0	0
5	9/15	Bills	Win, 45-3	0	1
6	9/22	Giants	Loss 3-26	2	0
10	9/26	Rams	Tie, 24-24	3	2
11	10/6	Bears	Win, 32-7	1	0
12	10/6	Saints	Win, 31-21	0	0
13	10/6	Rams	Loss, 13-16	3	1
14	10/6	Dolphins	Win, 27-13	4	1
15	10/6	Patriots	Win, 41-34	1	4
16	10/6	Seahawks	Loss, 13-42	1	0
17	10/6	Cardinals	Win, 27-13	1	0
Note: there's no week 18. Break btw regular season and postseason.					
19	10/6	Packers	Win, 45-31	1	1
20	10/6	Falcons	Win, 28-24	1	0
22	10/6	Ravens	Loss, 31-34	3	0

(a) (5 points) Based on this data, what is the overall probability that the 49ers will win a game this season?

Solution:

$$Pr(win) = \frac{9}{14} = 0.643$$

(b) (15 points) What is the conditional probability that the 49ers will win the next game given that Kaepernick is sacked in the current game? *Hint: there are only four pairs of sequential games in this data.*

Solution:

In the table below, we look at pairs of sequential games. For each pair, we assign three events: whether or not Kaepernick (1) was sacked in the first game, (2) fumbled in the first game, and (3) won the second game. Each event can either be true or false, and we use the table at the beginning of the question to determine. For example, in the first row, Kaepernick was not sacked in week 4 (number of sacks is 0, from the table of results above). He did not fumble in game 4 (number of fumbles is 0, see week 4 from table above). And they won in week 5 (see table above). So the events in the first row of the table below are F, F, T.

Weeks	Sacked This Game	Fumbled This Game	Win Next Game
4–5	F	F	T
5–6	F	T	F
10–11	T	T	T
11–12	T	F	T
13–14	T	T	T
14–15	T	T	T
15–16	T	T	F
16–17	T	F	T
17–19	T	F	T
19–20	T	T	T
20–22	T	F	F

$$Pr(win|sacked) = \frac{Pr(win \cap sacked)}{Pr(sacked)} = \frac{7/11}{9/11} = 0.7778$$

To get $Pr(win \cap sacked)$, look through the table to find rows that have T for both win and sacked. There are seven rows where Kaepernick was sacked in the current game and won the next. Divide by 11, the total number of rows in the table.

$$Pr(win \cap sacked) = 7/11$$

To get $Pr(sacked)$, look thru the table and find the number of rows where sacked is T. There are 9 of them. Divide by 11, the total numebr of rows.

$$Pr(sacked) = 9/11$$

(c) (15 points) What is the conditional probability that the 49ers will win the next game given that Kaepernick is sacked **and** he fumbles in the current game? *Hint: there are only four pairs of sequential games in this data.*

Solution:

$$Pr(win|\text{sacked} \cap \text{fumble}) = \frac{Pr(win \cap sacked \cap fumble)}{Pr(sacked \cap fumble)} = \frac{4/11}{5/11} = 0.8$$

(d) (20 points) Based on the info in this table, compute the probability that the 49ers will win the next game. *Hint: your calculation should be something like $Pr(win|sack$ and no fumble$)$*

Solution:

$$Pr(win|sacked \cap \text{no fumble}) = \frac{Pr(win \cap sacked \cap notfumble)}{Pr(sacked \cap notfumble)} = \frac{3/11}{4/11} = 0.75$$

(e) (10 points) Consider the following events:

E_1 The event that the 49ers win the next game

E_2 The event that Kaepernick fumbles in the current game

Is E_1 independent of E_2? Explain your reasoning with some math or a formula. *Hint: what is the definition of statistical independence?*

Solution: Statistical independence of E_3 and E_2 means that

$$Pr(E_1|E_2) = Pr(E_1)$$