CS 163 Discrete Math **Fall 2019**

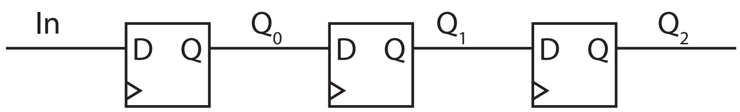
`http://neilklingensmith.com/teaching/loyola/cs163/`

Final A

Date: December 5, 2019

Name:

1. (10 points) Consider the shift register (LFSR) below. The input *In* can be either a 1 or 0, different on every clock cycle. We can think of the state as being the concatenation of the bits $Q_0Q_1Q_2Q_3$.



   (a) (10 points) Draw a state transition diagram.

2. (50 points) **Arithmetic on an 8-bit processor.** We have a really $#!tty 8-bit processor that only has an adder and a bit shifter. It has no ability to perform multiplication or division. We need to compute $(77_{10} - 22_{10}) \times 2$ using only addition and bit shifts.

(a) (15 points) First we're going to calculate the 2's complement representation of −22. In the box below, write out the binary representation of +22, then take its two's complement. Also convert the binary to hex in the boxes at right.

	Binary								Hex
+22									0x
1's(22)									0x
2's(22)									0x

(b) (15 points) Now add the two's complement of 22 to 77. The result should be the same as 77-22.

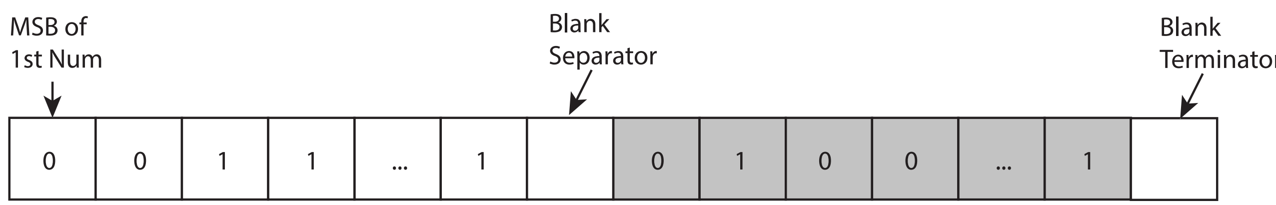
	Binary								Hex
2's(22)									0x
77_{10}									0x
2's(22) + 77_{10}									0x

(c) (10 points) Now multiply the result of the addition from part 2(b) by 2 using a bit shift.

	Binary								Hex
2's(22) + $77_{10} \times 2$									0x

(d) (10 points) Convert the result from part 2(c) to **decimal**.

3. (25 points) Design a finite state machine controller for a Turing Machine that overwrites one number with a second number.. Assume that the first number starts at the beginning of the tape and the second number is separated from the first by a blank space. In the diagram below of the input, the first number is shaded white and the second is shaded gray. Make sure you include a halt state that the controller goes to when it has completed its operation.



4. (25 points) Consider the following cyphertext encrypted with a shift cypher. Find the key and decrypt the message if you have time. *Note: I have added spaces every five letters to make the cyphertext more readable. Those spaces are not really part of the cyphertext.*

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NBSFS XQNYG XDROR SQRGK IQYSX QCSHD IPYEB QBKXN WKVOD KLSQY XOKXN LVOGW OYEDD
RONYY BDROG ROOVC MYEVN XDDKU OSDDR OOXQS XOPOV VKZKB DKVVL OMKEC OYPQB KXNWK
KXNRO BCEZO BCYXS MPKBD
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